1.5 - Investigating Quadrilaterals: Diagonals and Sides

Learning Goal:
• Determine the properties of different quadrilaterals by comparing slopes, lengths, and midpoints of diagonals and sides.

A **QUADRILATERAL** is a |

**TYPES:**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Parallelogram</td>
</tr>
<tr>
<td>2.</td>
<td>Rectangle</td>
</tr>
<tr>
<td>3.</td>
<td>Trapezoid</td>
</tr>
<tr>
<td>4.</td>
<td>Square</td>
</tr>
<tr>
<td>5.</td>
<td>Rhombus</td>
</tr>
</tbody>
</table>

What properties of the diagonals might we compare?

*length, slopes, midpoints*
\[ L_{CD} = \sqrt{50} \]
\[ L_{CF} = \sqrt{125} \]
\[ L_{FE} = \sqrt{50} \]
\[ L_{ED} = \sqrt{125} \]
\[ m_{CD} = \frac{1}{2} \]
\[ m_{CF} = 2 \]
\[ m_{FE} = \frac{1}{2} \]
\[ m_{ED} = 2 \]

\[ L_{EC} = 9.22 \]
\[ L_{FD} = 16.38 \]
\[ M_{EC} = (7, 3.5) \]
\[ M_{FD} = (7, 3.5) \]
\[ m_{Ec} = -\frac{9}{2} \]
\[ m_{ED} = \frac{11}{12} \]

\[ L_{AB} = 6 \]
\[ L_{BC} = 11 \]
\[ L_{DC} = 6 \]
\[ L_{DA} = 11 \]
\[ m_{DA} = \text{undefined} \]
\[ m_{DC} = 0 \]

\[ L_{AC} = \sqrt{157} \]
\[ m_{AC} = \frac{11}{6} \]
\[ L_{BD} = \sqrt{157} \]
\[ m_{BD} = \frac{11}{6} \]
\[ M_{AC} = (-2, 2.5) \]
\[ M_{BD} = (-2, 2.5) \]
**Diagonals and Sides**

- **M\(\overline{AC}\) = (7, 6)**
- **M\(\overline{BD}\) = (4, 6)**
- **L\(\overline{AC}\) = \(\sqrt{80}\)**
- **L\(\overline{BD}\) = \(\sqrt{80}\)**
- **M\(\overline{AC}\) = \(-\frac{4}{3}\)**
- **M\(\overline{BD}\) = \(\frac{1}{2}\)**

**Quadrilateral with Vertices**

- \(L\overline{CB} = 5\), \(m\overline{CB} = 0\)
- \(L\overline{BA} = 5\), \(m\overline{BA} = -\frac{4}{3}\)
- \(L\overline{DA} = 11\), \(m\overline{DA} = 0\)
- \(L\overline{DC} = 5\), \(m\overline{DC} = \frac{4}{3}\)

**Diagonal Ratios**

- **M\(\overline{CE}\) = \(\sqrt{36}\) units**
- **M\(\overline{FD}\) = \(\sqrt{36}\) units**
- **M\(\overline{FD}\) = (4, 5)**
- **M\(\overline{CE}\) = (4, 5)**
- **M\(\overline{CE}\) = \(\frac{8}{0}\) = undefined**
- **M\(\overline{FD}\) = 0**

**Diagonal Lengths**

- \(L\overline{CE} = 6\) units
- \(L\overline{FD} = 6\) units

**Diagonal Angles**

- \(\angle CDE = 90^\circ\)
- \(\angle ECF = 90^\circ\)
- \(\angle EDF = 90^\circ\)
- \(\angle CEF = 90^\circ\)

**Diagonal Relationships**

- By \(N\overline{CF}\)
  - \(N\overline{CF}\) operations
CLASSIFYING QUADRILATERALS

<table>
<thead>
<tr>
<th>Type of Quadrilateral</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>Same length not ( \frac{1}{2} ) bisect each other -2 sets of // sides -2 sets of ( \perp ) sides</td>
</tr>
<tr>
<td>Square</td>
<td>Same length - all sides = ( \frac{1}{2} ) bisect each other -2 sets //</td>
</tr>
<tr>
<td>Parallelogram</td>
<td>bisect each other -2 sets // -2 sets of ( \perp ) sides</td>
</tr>
<tr>
<td>Rhombus</td>
<td>bisect each other perpendicular - all sides = -2 sets of //</td>
</tr>
<tr>
<td>Trapezoid</td>
<td>do not bisect each other - one set of // sides</td>
</tr>
</tbody>
</table>
Homework

Investigate the properties of the sides of the different quadrilaterals using the shapes from Investigation 1.

1.5 continued

Investigating Quadrilaterals: Diagonals and Sides

Learning Goal:
• Classify the type of quadrilateral given, using properties of their diagonals and sides
Example 1
Determine the type of quadrilateral having vertices A(-5, 4), B(-2, 8), C(6, 2), and D(3, -2).

\[ \frac{\overline{AC}}{\overline{BD}} \]
\[ \overline{AC} = \overline{BD} \]
\[ \overline{AC} = \sqrt{125} \]
\[ \overline{BD} = \sqrt{125} \]
\[ M_{\overline{AC}} = (0.5, 3) \]
\[ M_{\overline{BD}} = (0.5, 3) \]
\[ m_{\overline{AC}} = \frac{-2}{11} \]
\[ m_{\overline{BD}} = \frac{-10}{5} \]
\[ = -2 \]

Example 2
A quadrilateral has vertices at O(0,0), P(3,5), Q(8,6), and R(5,1). Verify that OPQR is a parallelogram.

If parallelogram, diagonals must bisect each other:

\[ M_{\overline{OQ}} = (4, 3) \]
\[ M_{\overline{PR}} = (4, 3) \]
\[ \frac{5-0}{3-0} = \frac{5}{3} \]
\[ \frac{6-5}{8-3} = \frac{1}{5} \]
\[ \Rightarrow \text{parallel} \]
\[ \frac{5+1}{5+5} = \frac{6}{10} = \frac{3}{5} \]
\[ \Rightarrow \text{parallel} \]

\[ \therefore \text{we have 2 sets of parallel sides, OPQR is a parallelogram} \]
Example 3
Classify the quadrilateral with vertices E(2,3), F(-1,1), G(-2,-9), and H(-2,1).

Classify TWO of the following quadrilaterals:

a) A(-4,3), B(-1,4), C(10,1), and D(-5,-4)
b) K(-4,1), L(-1,3), M(3,-3), and N(0,-5)
c) Q(3,4), R(1,1), S(4,-4), and T(6,-1)
d) W(-2,1), X(2,3), Y(4,-1), and Z(0,-3)

Use diagonals for ONE and sides for the OTHER
HOMEWORK: Page 105 #7, 10bc, 11ab